Excitation of spin waves by a spin polarized current.

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Abstract

Numerical and analytical analysis is used to explain a recently observed experimental phenomenon – excitation of spin waves in a spin valve due to an applied spin polarized current. Various excited spin waves are identified and a stability analysis is used to identify different excitation regimes as a function of the of applied current.

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In the emerging field of spintronics,[1] the operation of various devices involves an interaction with a spin polarized current. In particular, one can use a spin polarized current to produce magnetization reversal, as opposed to conventional magnetic memory devices which are controlled by external magnetic fields. While the importance of spin waves in operating spin valves has been discussed, the excitation of spin waves by a spin polarized current has only recently have been observed by Kiselev et al.[2] In this experiment, a complex oscillation of the magnetic moments was induced in the sample, even through the current strength was below that required for the reversal. In a later article Lee et. al.[3] explained that these oscillations correspond to the excitation of inhomogeneous spin waves by the applied spin polarized current. However, the modes that are excited have not been identified and a detailed analysis of their excitation has yet to be performed. In the present work we specifically address these issues.

We recall the Landau-Lifshitz[4] equation in the presence of a dissipative Gilbert term and a spin transfer torque[5,6,7,8,9] :

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{h} ; \qquad (1)$$

where

$$\mathbf{h} = \mathbf{h}^{\mathrm{true}} + \frac{\mathbf{m}}{M_s} \times \left(\beta \mathbf{h}^{\mathrm{true}} - I\mathbf{h}^{J}\right); \qquad (2)$$

here $\mathbf{m}$ is the magnetic moment and γ is the gyromagnetic ratio. The combined effects of dissipation and a spin polarized current are modeled by the term $\frac{\mathbf{m}}{M_s} \times \left(\beta \mathbf{h}^{\mathrm{true}} - I\mathbf{h}^{J}\right)$, where β is a parameter governing the dissipation, $\mathbf{h}^J$ is the polarization of the current, and I is an empirical factor measuring the strength of the coupling (in units of magnetic field where 1000 Oe corresponds to 10^8 A/cm^2).

A typical spin valve consists of two magnetic layers, separated by a nonmagnetic layer. One of the layers is assumed to have a constant magnetization, and the other, the so-called free layer, has a magnetic moment that can change with time. Initially both layers are magnetized along the same direction; the spin polarized current entering the free layer is also polarized along the same direction.

Here we will use dimensions similar to those reported by Kiselev et al[2,10] in which the first (constant magnetization) layer is 20nm thick and the free layer is 2nm thick. The layers are deposited from cobalt and approximate an ellipse with principal diameters 130 by 70nm. We assume an exchange stiffness $A = 1.3 \cdot 10^{-6}$ erg/cm and a saturation magnetization $M_s = 795\,\mathrm{emu/cm^3}$; it is assumed that there is no exchange interaction between the layers. Our value of M_s is quite different from that was used by Kiselev[4] to fit the frequency of the uniform mode. Using either our numerical eigenvalue algorithm (see below) or the Kittel equation (with numerically calculated demagnetization tensor components) we find that the value we have adopted for M_s is more consistent with the experimental results.

Our coordinate system is chosen such that the easy axis is parallel to z with the magnetic layers lying in the y-z plane. The current direction is along x and it is polarized along z. In our case a current of 1 mA should correspond to $a_J \cong 144$ Oe, and the damping coefficient β is taken as[4] 0.014. We will consider the case when the external magnetic field cancels the magnetic field of the thick layer.

We start by linearizing the Landau-Lifshitz equation with respect to a small time-dependent oscillating part of the magnetization, $\mathbf{m}_i^{(1)}(t) = \mathbf{m}_i^{(1)} e^{-i\omega t}$, and corresponding oscillating magnetic fields, $\mathbf{h}_i^{(1)}(t)$. We assume that initially the system is in equilibrium with the magnetic moments $\mathbf{m}_\mathrm{i}^{(0)}$ aligned parallel to the local magnetic fields, $\mathbf{h}_\mathrm{i}^{(0)}$. In the absence of a current, we have the following eigenvalue equation[11]:

$$-\frac{\mathrm{d}\mathbf{m}_\mathrm{i}^{(1)}}{\mathrm{dt}} = \gamma\left[\mathbf{m}_\mathrm{i}^{(0)} \times \mathbf{h}_\mathrm{i}^{(1)} + \mathbf{m}_\mathrm{i}^{(1)} \times \mathbf{h}_\mathrm{i}^{(0)}\right] + \frac{\gamma\beta}{\mathrm{M_s}} \mathbf{m}_\mathrm{i}^{(0)} \times \left[\mathbf{m}_\mathrm{i}^{(0)} \times \mathbf{h}_\mathrm{i}^{(1)} + \mathbf{m}_\mathrm{i}^{(1)} \times \mathbf{h}_\mathrm{i}^{(0)}\right] \quad \text{(3a)}$$

which we rewrite as

$$\mathrm{i}\omega_\mathrm{k} \mathbf{V}_\mathrm{i}^{(\mathrm{k})} = \gamma \mathbf{B}_\mathrm{ij} \mathbf{V}_\mathrm{j}^{(\mathrm{k})} \quad \text{(3b)}$$

where ω_k are the eigenfrequencies, $\mathbf{V}_\mathrm{i}^{(\mathrm{k})}$ and $\mathbf{V}_\mathrm{iL}^{(\mathrm{k})}$ are respectively the right and left eigenvectors (giving the amplitude and phase of the precession of each moment for the mode ω_k), and $\mathbf{B}_\mathrm{ij}$ is a matrix which depends on the equilibrium structure and various parameters controlling the motion. Solving Eq. (3) provides us with the complete spin wave spectrum for a given level of discretization. It should be noted that the matrix given by Eq.(3) is not symmetric; therefore it can be shown that left and right eigenvectors are orthogonal to each other, but, in general, not among themselves.

We now examine what happens if a spin polarized current is applied to the sample. In this case Eq. (3) transforms into an *inhomogeneous* equation involving a source term generated by the applied current:

$$\frac{\mathrm{d}\mathbf{m}_\mathrm{i}^{(1)}}{\mathrm{dt}} + \gamma\left[\mathbf{m}_\mathrm{i}^{(1)} \times \mathbf{h}_\mathrm{i}^{(0)} + \mathbf{m}_\mathrm{i}^{(0)} \times \mathbf{h}_\mathrm{i}^{(1)}\right] + \frac{\gamma\beta}{\mathrm{M_s}} \mathbf{m}_\mathrm{i}^{(0)} \times \left[\mathbf{m}_\mathrm{i}^{(0)} \times \mathbf{h}_\mathrm{i}^{(1)} + \mathbf{m}_\mathrm{i}^{(1)} \times \mathbf{h}_\mathrm{i}^{(0)}\right] \approx -\gamma \mathbf{g}_\mathrm{i}(\mathrm{t})$$

(4a)

where $\mathbf{g}_i(t)$ is the source term given by:

$$\mathbf{g}_i(t) = -\frac{I}{M_s}\mathbf{m}_i^{(0)} \times (\mathbf{m}_i^{(0)} \times \mathbf{h}^J) - \frac{I}{M_s}\mathbf{m}_i^{(0)} \times (\mathbf{m}_i^{(1)} \times \mathbf{h}^J) - \frac{I}{M_s}\mathbf{m}_i^{(1)} \times (\mathbf{m}_i^{(1)} \times \mathbf{h}^J);$$

(4b)

that is a source term responsible for the excitation of spin waves. In obtaining these equations we neglected terms like $\frac{I}{M_s}\mathbf{m}^{(1)} \times (\mathbf{m}^{(0)} \times \mathbf{h}^J)$, which can be shown to not contribute to the dynamics of the spin waves.

The solutions of the above equations consist of two parts: a solution of the homogeneous Eq. (3a,b), and a solution of the inhomogeneous Eq. (4a,b). In general, the sum of these solutions would be chosen to satisfy any initial conditions. However, since we are mostly interested in steady-state solutions, we will not consider such general solutions; in the presence of damping ($\beta > 0$) the solutions to the homogeneous equation decay in time and do not contribute to the steady state behavior.

While detailed mathematical solutions of Eq. (4) were studied in reference[12], here we will use an alternate method of analysis, first introduced in reference[13]. Since the left and right eigenvectors of the homogeneous Eq. (3b) form a complete set (that can be used to describe an arbitrary magnetic configuration at some point in time), the solution of Eq. (4) can be expanded in terms of them as

$$\mathbf{m}_i^{(1)} = \sum_k a_k(t)\mathbf{V}_i^{(k)} . \tag{5}$$

In this case Eq. (4) becomes:

$$\frac{da_k(t)}{dt} + ia_k(t)\omega_k = -\gamma \iiint \mathbf{g}\mathbf{V}_L^{*(k)} dxdydz; \qquad (6)$$

in a discrete version, the integration is replaced by a summation. Starting with the first contribution to the source term in Eq. (4b) we obtain

$$\frac{da_k(t)}{dt} + ia_k(t)\omega_k = \gamma \frac{I}{M_s} \sum_i \left(\mathbf{m}_i^{(0)} \times (\mathbf{m}_i^{(0)} \times \mathbf{h}^J) \right) \cdot \mathbf{V}_{iL}^{*(k)} \qquad (7a)$$

that can be solved for $a_k(t)$

$$a_k(t) = \gamma I \frac{\sum_i \left(\mathbf{m}_i^{(0)} \times (\mathbf{m}_i^{(0)} \times \mathbf{h}^J) \right) \cdot \mathbf{V}_{iL}^{*(k)}}{i\omega_k M_s}. \qquad (7b)$$

We see that in this case the solution is time independent and simply represents a static shift from the initial equilibrium; here a_k is nonzero only for the “modes” that are concentrated in the regions where the equilibrium magnetization is not parallel to the current polarization and therefore is not parallel to the external field. The modes excited by this term are so-called “edge modes” corresponding to a static shift between the new equilibrium configuration, formed in the presence of the current, and the old equilibrium.

The second term in Eq. (4) gives

$$\frac{da_k(t)}{dt} + ia_k(t)\omega_k = \gamma \frac{I}{M_s} \sum_{i,k'} a_{k'}(t) \left(\mathbf{m}_i^{(0)} \times (\mathbf{V}_i^{(k')} \times \mathbf{h}^J) \right) \cdot \mathbf{V}_{iL}^{*(k)} \qquad (8)$$

which we rewrite as

$$\frac{da_k(t)}{dt} + ia_k(t)\omega_k = \gamma \sum_{k'} c_{kk'} a_{k'}(t) \qquad (9a)$$

with

$$c_{kk'} = \frac{I}{M_s} \sum_i \mathbf{V}_i^{(k')} \mathbf{V}_{iL}^{*(k)} \left(\mathbf{m}_i^{(0)} \bullet \mathbf{h}^J \right). \qquad (9b)$$

If all of the magnetic moments are parallel to each other, $c_{kk'} = I\frac{m_z^{(0)}}{M_s}\delta_{kk'}$; this is approximately the case since the polarization of the current is aligned with both the easy axis and the external field; if k' is not equal to k, $c_{kk'}$ can initially be only a few percent of $I\frac{m_z^{(0)}}{M_s}$. For such times we can assume that $c_{kk'} = I\frac{m_z^{(0)}}{M_s}\delta_{kk'}$ yielding

$$\frac{da_k(t)}{dt} + ia_k(t)\omega_k = \gamma I\frac{m_z^{(0)}}{M_s}a_k(t) \tag{10}$$

which has the solution

$$a_k(t) = a_k(0)e^{-i\omega_k t}e^{\gamma I\frac{m_z^{(0)}}{M_s}t}. \tag{11}$$

Since $\omega_k = \omega_k{}' - i\omega_k{}''$ we can rewrite Eq. 11 as

$$a_k(t) = a_k(0)e^{-i\omega_k{}'t}e^{\left(\gamma I\frac{m_z^{(0)}}{M_s} - \omega_k{}''\right)t}. \tag{12}$$

Note there are two regimes i) if $\gamma I\frac{m_z^{(0)}}{M_s} < \omega_k{}''$, the oscillations die out exponentially in time, and ii) if $\gamma I\frac{m_z^{(0)}}{M_s} > \omega_k{}''$, the excitation of k^{th} mode increases exponentially, and the initial equilibrium therefore becomes unstable.

The uniform mode typically has the lowest frequency, for both its real and imaginary parts. Because of this, the uniform mode is expected to be the most strongly excited mode; for larger currents higher frequency modes will be excited, however their growth rate will be smaller than that of the uniform mode. The phase of the oscillations is fixed by $a_k(0)$, which in our model is fixed by (small) random component of the orientations which are present in the initial state and simulate finite temperature effects.

In order to further study the excitation of the uniform mode we can use a macrospin model, shape effects being included through a demagnetization tensor $A_{\alpha\beta}$:

$$H_\alpha = \sum_\beta A_{\alpha\beta} M_\beta \, . \tag{13}$$

For the above-mentioned ellipse ($130 \times 70\text{nm}$), the numerically-estimated principal elements are $A_{zz} = -0.27$, $A_{yy} = -0.61$, and $A_{xx} = -11.69$. Since our numerical studies show that the x component of the magnetization remains small, we will neglect $m_x^{(0)}$ in what immediately follows. Modifying the Kittel formula to account for damping one then has:

$$\omega = \gamma\sqrt{\left(H_0 + m_z^{(0)}B\right)\left(H_0 + m_z^{(0)}A\right)} - i\gamma\frac{\beta}{M_s}m_z^{(0)}\left(\frac{\left(H_0 + m_z^{(0)}B\right) + \left(H_0 + m_z^{(0)}A\right)}{2}\right) \tag{14}$$

where we defined

$$\begin{aligned} A &\equiv \left(A_z - A_y\right) = -0.27 + 0.61 = 0.34 \\ B &\equiv \left(A_z - A_x\right) = -0.27 + 11.69 = 11.42 \\ C &\equiv \left(A_y - A_x\right) = -0.61 + 11.69 = 11.08. \end{aligned} \tag{15}$$

In the absence of an external field, Eq. (14) can be extended to the case of an arbitrary direction of the magnetization in z-y plane as

$$\omega = \gamma\sqrt{(m_z^{(0)})^2 AB - (m_y^{(0)})^2 AC} - i\gamma\frac{\beta}{2M_s}\left((m_z^{(0)})^2\left(A + B\right) + (m_y^{(0)})^2\left(C - A\right)\right). \tag{16}$$

Inserting this expression into Eq. (12) we see that a steady state solution is possible (as first predicted by Li and Zhang[10]); even though current is larger than the imaginary part of the uniform mode frequency at t = 0 the, for a certain value of $m_z^{(0)}$ the following expression is satisfied:

$$\gamma I_c \frac{m_z^{(0)}}{M_s} = \omega_k'' \tag{17a}$$

and on using Eq. (16) we have

$$\frac{\beta}{2}\left(m_z^{(0)^2} \left(A + B\right) + m_y^{(0)^2} \left(C - A\right)\right) = I m_z^{(0)}; \tag{17b}$$

i.e., the uniform mode simply oscillates with its own resonant frequency.

However, this is not the only possible behavior. Since the eigenvector depends on the direction of magnetization, it is clear that initially ($m_y^{(0)} = 0$) the oscillations are confined to the x-y plane; excitation of the uniform mode leads to a slow decrease of $m_z^{(0)}$. However when $m_z^{(0)}$ is small the oscillations include a significant z component. Because of this, even if the system reaches the value of $m_z^{(0)}$ which satisfies Eq. (17), the oscillations in z-y plane do not stop. If the current is not sufficiently strong, it is possible for the magnetization to move towards smaller values of $m_z^{(0)}$, where the oscillations increase, followed by the magnetization moving towards larger of values of $m_z^{(0)}$ where the damping exceeds the excitation due to the current, and so on, back and forth, producing a complex oscillatory pattern. The non-linear interactions due to $\frac{I}{M_s}\mathbf{m}^{(1)} \times (\mathbf{m}^{(1)} \times \mathbf{h}^J)$ and other nonlinear terms should also be taken into account.

However, if the current is strong enough, and therefore the value of $m_z^{(0)}$ for which the damping exceeds the excitation is small enough, the value of $m_z^{(0)}$ can become negative[8]. As soon as this happens, the term in Eq. (12) arising from the applied current changes sign and produces additional damping. The more negative $m_z^{(0)}$ is, the higher the damping is, and, as a result, the system switches to the new equilibrium configuration, with the magnetization aligned in the opposite direction with respect to the initial configuration

Since we have made a number of approximations in the above discussion, we need to verify our conclusions with numerical experiments. We used a Runge-Kutta based simulation that proceeds as follows: We first obtain the equilibrium configuration in the absence of the current. We then slightly perturb all the magnetic moments in a random manner, thereby simulating a finite temperature excitation in the system, while at the same time applying the spin polarized current. This is followed by taking a Fourier transform of time dependence of M_z, which corresponds to experimentally measuring the sample resistance[4]. However, Fourier analysis of the magnetization is not sufficient to tell us which modes are actually excited –growth or decay broadens the spectrum; additional effects that we have not addressed can also affect the spectrum. In order to have a clearer picture of which modes are excited we will *spatially decompose* the oscillations into small amplitude spin waves:

$$\begin{aligned} &\mathbf{m}^{(1)}(t) = \mathbf{m}(t) - \mathbf{m}^{(0)} \\ &a_k(t) = \iiint \mathbf{m}^{(1)}(t) \cdot \mathbf{V}_L^{*(k)} dxdydz \end{aligned} \tag{19}$$

It should be noted that such expansion onto a set of eigenvectors gives us a somewhat incomplete representation: since only two coordinates per dipole are being used, we are failing to account for the sign of the projection of $\mathbf{m}^{(1)}(t)$ onto initial configuration $\mathbf{m}^{(0)}$. In the current work this problem can be neglected since once the sign of the magnetization is changed the oscillations are going to be damped out.

We start with a "damped" regime, in which most of the oscillations die out according to Eq. (12). Initially we see a small excitation of nearly all the modes, which can be explained by the fact that the momentarily applied current possesses a broad Fourier spectrum, which therefore excites a broad spectrum of modes. Following this, all modes decay exponentially, with the exception of the modes identified from Eq. (7). The numerically calculated value of the threshold current is in a good agreement with the theory, being typically ~ 0.04 mA higher than that given by Eqs. (12) and (14). For the currents above this value (0.49 mA with no external field, 0.71 mA for $H_0 = 2600$ Oe) we indeed see a steady-state behavior, as shown in Fig.1. In addition to the uniform mode (f=4.09 GHz) we also show behavior of two non-uniform modes with nieghbouring frequencies: 5.23 and 8.89 GHz. As noted above, the oscillations of z component of magnetization are determined mostly by z component of the eigenvector (Fig. 2). It is interesting to note that if instead of starting with a random initial perturbation, we start with a finite amplitude of the uniform mode, the system still evolves into a steady state that involves non-uniform modes; the only difference will be that it will take far less time for a uniform mode to reach the steady state regime. The reasons behind this are the neglect mode-mode coupling terms in Eq. (9) and other non-linear terms. If the current is increased, a complex oscillation pattern forms, involving both oscillations of the uniform mode (Fig.3) and the z component of the magnetization (Fig. 4). If the current is increased even further, switching occurs (Figs. 5 and 6). As can be seen, the general behavior is consistent with the above analytical analysis – the uniform mode grows until a semi-steady state forms for a very small value of M_z; then, due to non-zero value of V_z the system acquires a small but negative value of M_z. As soon as this happens, the modes are quickly suppressed by the combined effects of the damping and the applied current. Fig. 7 shows the relative maximum excitation of modes for different currents, corresponding respectively to the damped, steady state, and oscillatory regimes. As one can see, in a damped regime (I = 0.28 mA) excitation of the modes is rather non-selective, and is determined by the coupling between initial random excitations and the momentarily applied current. In the steady state regime (I = 0.5 mA) only a very few low frequency modes satisfy Eq. (12) and as a result, only few modes are excited. In an oscillatory and switching regimes (I = 0.7 mA and 0.76 mA), the current is strong enough to excite many modes; however, low frequency modes are excited more strongly than higher frequency ones – the exponent in Eq. (12) is typically larger for lower than higher frequency modes.

Conclusions

In our work we succeeded in establishing that there are four possible spin wave excitation regimes in the presence of an applied spin polarized current. Depending on the magnitude of the current we can have:

a. A damped regime where the system approaches a new static equilibrium.
b. A steady-state regime, characterized by a nearly constant excitation of a very few low frequency modes, and relatively small excitation of other modes.
c. Oscillatory behavior, where a broad spectrum of modes is excited, with the preference given to low frequency modes.
d. Switching, where the semi-steady state excitation of modes is superseded by very fast damping into the new static equilibrium, with the magnetization anti-parallel to its initial value.

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Figures.

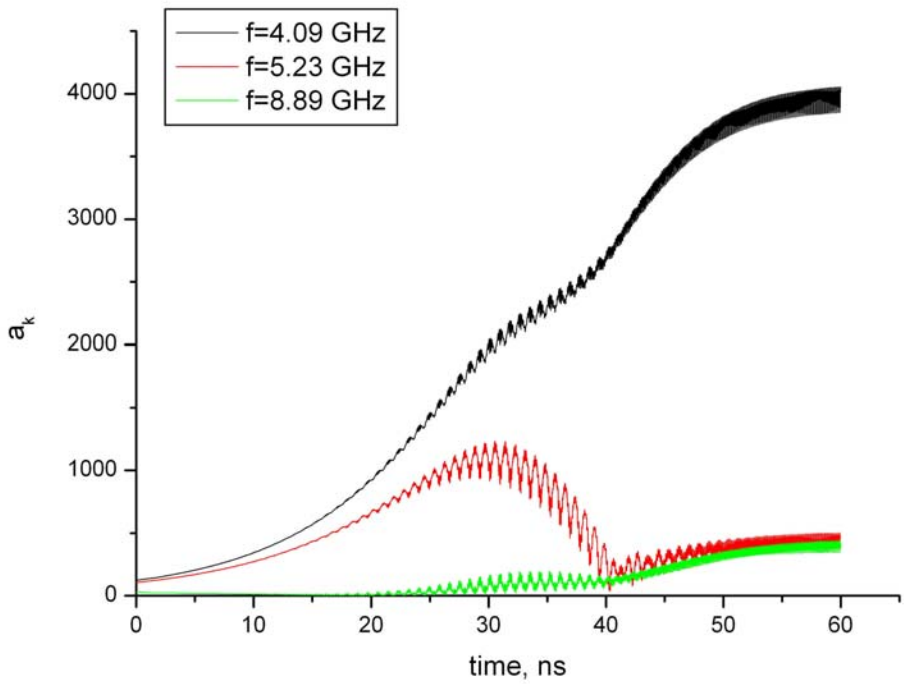

Figure 1. Amplitude of the most exited modes as a function of time with no external field and a current of 0.5 mA.

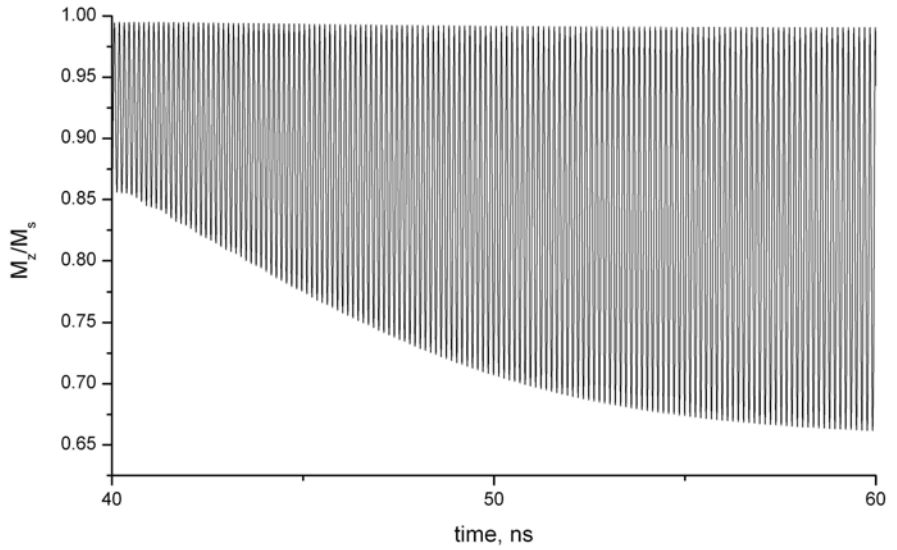

Figure 2. Z component of the magnetization as a function of time with no external field and a current of 0.5 mA.

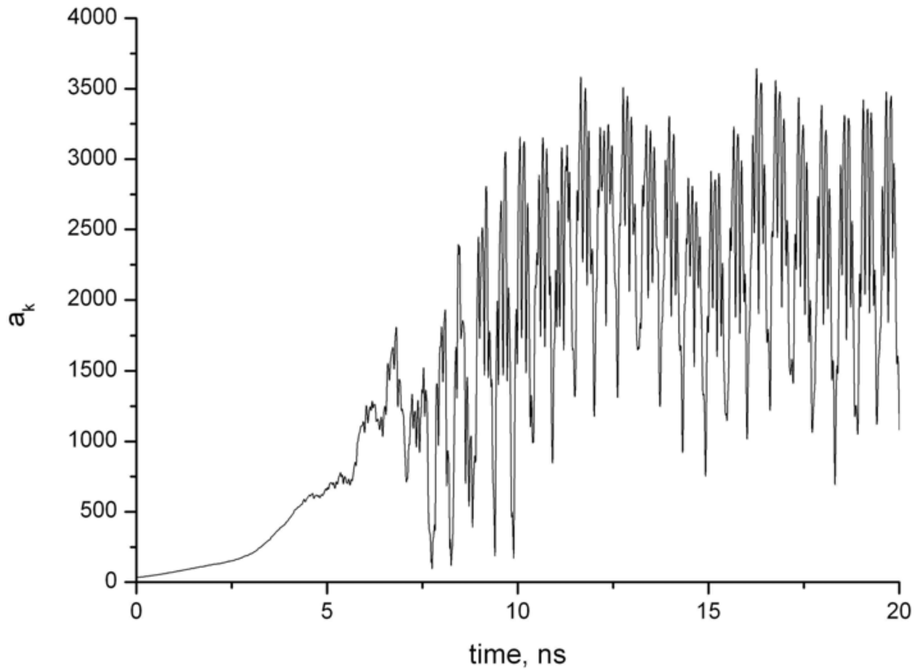

Figure 3. Amplitude of the uniform mode as a function of time with no external field and a current of 0.7 mA.

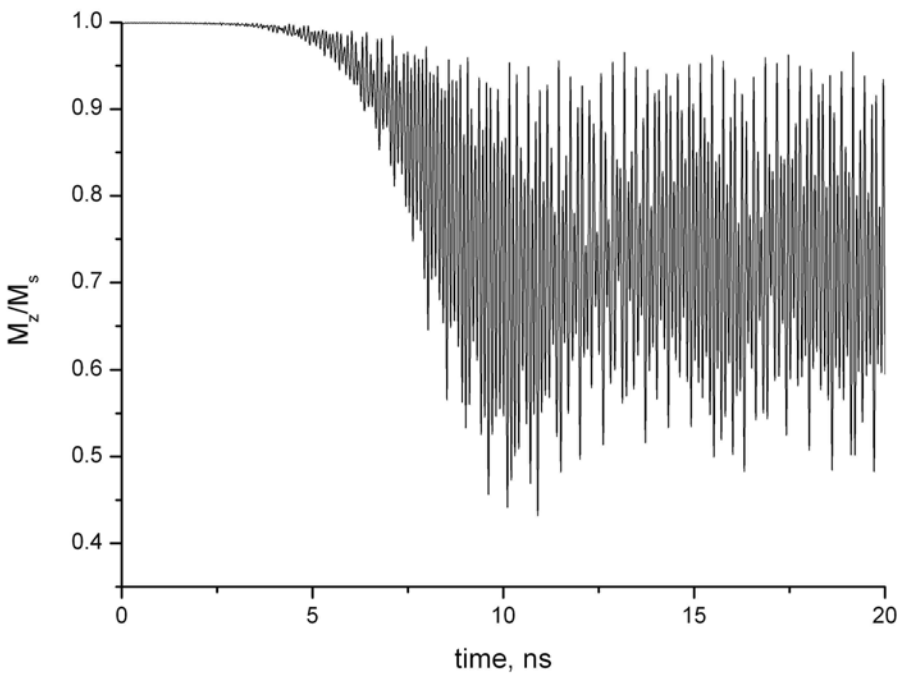

Figure 4. Z component of the sample's magnetization as a function of time with no external field and a currentof 0.7 mA.

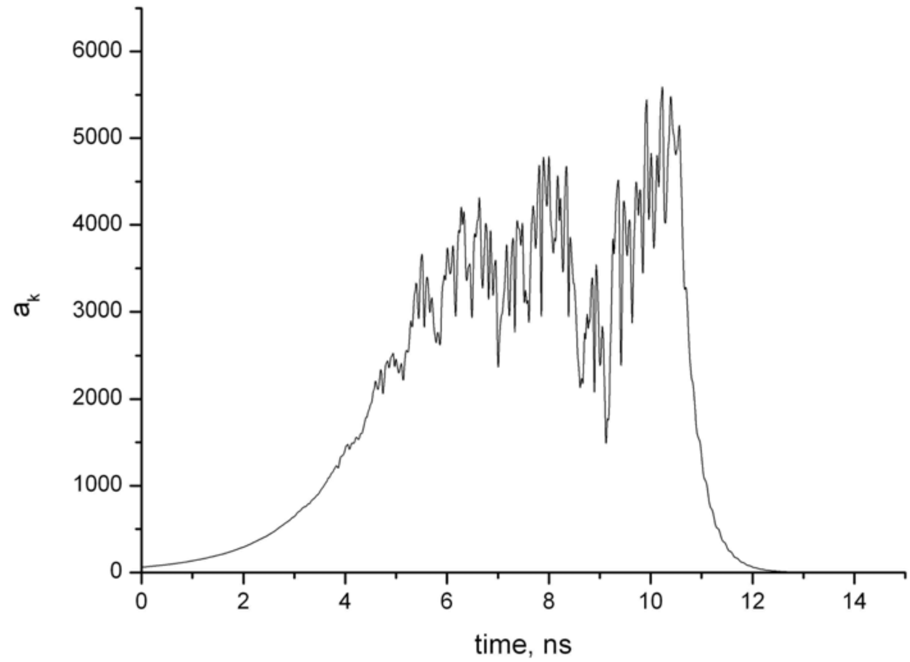

Figure 5. Amplitude of the uniform mode as a function of time with no external field and a current of 0.76 mA.

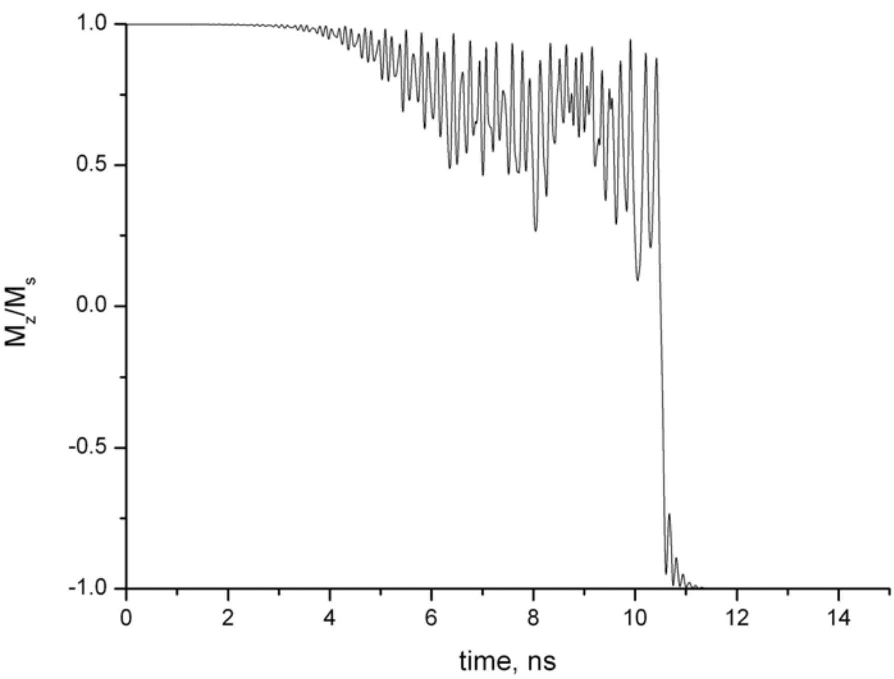

Figure 6. Z component of the sample's magnetization as a function of time with no external field and a current of 0.76 mA.

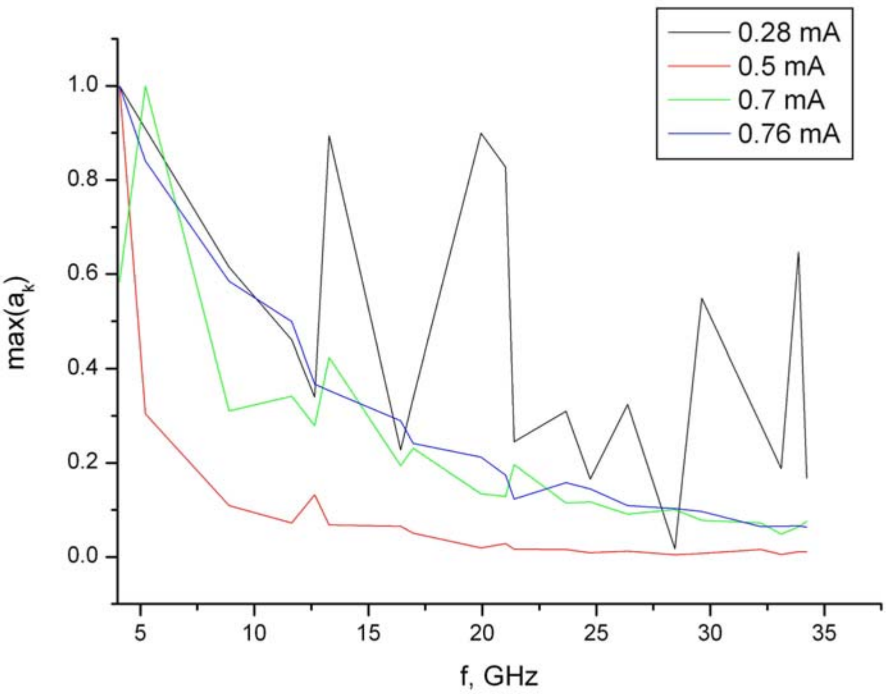

Figure 7. Maximum amplitude of various modes as a function of their frequency for different current values and no external field.