Microscopic study of magnetostatic spin waves

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(Presented on 8 November 2004; published online 6 May 2005)

A relatively new method is developed to numerically calculate the spin-wave-related properties of a magnetic body of arbitrary shape. Starting with a discrete dipole approximation and the linearized Landau–Lifshitz equation, the resonant frequencies and the associated amplitudes of the individual moments are obtained for all modes; from this information we are able to calculate the energy absorbed by the various modes excited by a position- and time-dependent external magnetic field. The method has been demonstrated for a number of cases including thin disks and rings and for equilibrium configurations ranging from the saturated high-field limit to the vortex states at low fields. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1851873]

I. INTRODUCTION

Over the past few years the subject of resonant spin waves in vortex structures in magnetic nanoparticles has attracted much attention. While significant progress has been made, in both the experimental and theoretical aspects, we here stress that the implementation of an uncommonly used method based on the representation of a magnetic body by discrete dipoles, together with a linearization of the associated dynamical equations motion, can be quite powerful.

Very similar methods have been tried in the past by Politi *et al.*[1] and by Nortemann *et al.*;[2] however, in their original form these methods were of limited applicability, lacking among other things a procedure that would allow one to calculate the absorption due to arbitrary external time-dependent magnetic fields. In the current paper we implement our version of the above-mentioned method (for more details, see Ref. 3) and use it to study resonant modes of a typical vortex structure consisting of a central core, where the magnetization points primarily out of plane, and the rest of the sample, where the magnetization is in plane and encircles the vortex core (as studied, e.g., by Usov and Peschany[4,5]).

Recent work by Hertel and Kirschner[6] and by Park *et al.*[7] employed, respectively, finite elements and direct integration of the Landau–Lifshitz equation in order to obtain a theoretical description of resonant spin waves. Unlike these authors, we are not able to treat the nonlinear response of magnetic media, and in the present case we are not able to study the precession of the vortex core as a whole, in the way described by Guslienko *et al.*,[8] but nevertheless our method is capable of producing the spin-wave spectrum and yielding a reliable representation of absorption spectra.

II. NUMERICAL METHOD EMPLOYED

We start with the Landau–Lifshitz equation with a dissipative Gilbert term

$$\frac{d\mathbf{m}_i}{dt} = -\gamma \mathbf{m}_i \mathbf{h}_i^{\text{total}} - \frac{\beta\gamma}{M_s}\mathbf{m}_i(\mathbf{m}_i\mathbf{h}_i^{\text{total}}), \tag{2.1}$$

where $\mathbf{m}_i$ is the magnetic moment of the ith spin, γ is the gyromagnetic ratio, and β is a parameter governing the dissipation. We will assume that the applied fields and magnetic moments can be written as the sum of a zeroth-order static part and a small first-order time-dependent perturbation

$$\mathbf{m}_i = \mathbf{m}_i^{(0)} + \mathbf{m}_i^{(1)}(t), \tag{2.2a}$$

$$\mathbf{h}_i = \mathbf{h}_i^{(0)} + \mathbf{h}_i^{(1)}(t). \tag{2.2b}$$

Assuming the time-dependent part has the form

$$\mathbf{m}_i^{(1)}(t) = \mathbf{m}_i^{(1)} e^{-i\omega t}, \tag{2.3}$$

the linearized form of Eq. (2.1) is given by

$$i\omega \mathbf{m}_i^{(1)} = \gamma[\mathbf{m}_i^{(0)}\mathbf{h}_i^{(1)} + \mathbf{m}_i^{(1)}\mathbf{h}_i^{(0)}] + \frac{\gamma\beta}{M_s}\mathbf{m}_i^{(0)}[\mathbf{m}_i^{(0)}\mathbf{h}_i^{(1)} + \mathbf{m}_i^{(1)}\mathbf{h}_i^{(0)}]; \tag{2.4}$$

the eigenvalues, ω, characterize the normal modes. For $\beta = 0$, corresponding to the dissipationless case, ω will be real, provided the zeroth-order spin directions have been relaxed to their stable equilibrium directions. Due to the fact that we use a complex form to represent the physical moments, which are real, any solution of Eq. (2.3) with a positive ω' and eigenvector $\mathbf{m}_i^{(1)}$ will be accompanied by a second solution with eigenfrequency $-\omega'$ and eigenvector $\mathbf{m}_i^{(1)*}$.

The microscopic fields entering Eq. (2.3) include the dipole-dipole and exchange interactions

$$\mathbf{h}_i^{(0)} = \mathbf{h}_i^{(0)\text{dipole}} + \mathbf{h}_i^{(0)\text{exchange}} + \mathbf{H}_0 = \sum_{j\neq i} \mathbf{m}_j^{(0)}\left[\nabla\nabla\left(\frac{1}{\mathbf{r}_{ij}}\right)\right] + \mathbf{h}_i^{(0)\text{exchange}} + \mathbf{H}_0, \tag{2.5}$$

and

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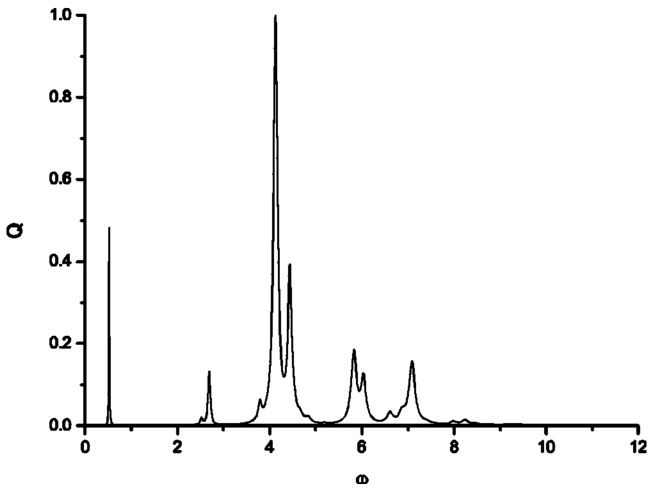

FIG. 1. Absorption as a function of ω for a uniform field parallel to x direction, $J=5$.

$$\mathbf{h}_i^{(1)} = \mathbf{h}_i^{(1)\text{dipole}} + \mathbf{h}_i^{(1)\text{exchange}} = \sum_{j\neq i} \mathbf{m}_j^{(1)} \left[\nabla \nabla \left(\frac{1}{\mathbf{r}_{ij}} \right) \right] + \mathbf{h}_i^{(1)\text{exchange}}; \quad (2.6)$$

it is straightforward to add anisotropy energy terms. In the present paper, we restrict the exchange interaction to the nearest neighbors only, i.e.,

$$\mathbf{H}_{\text{exchange}} = J \sum_{i}^{nn} \mathbf{m}_i. \quad (2.7)$$

After solving Eq. (2.3) for the eigenvectors $V_i^{(k)}$ and the corresponding eigenvalues $\omega^{(k)}$, we must then ask how the various modes of oscillation can be excited by applying a time-dependent external magnetic field. Following the standard procedure for solving an inhomogeneous linear first-order ordinary differential equation (ODE), in the presence of this dynamic field, Eq. (2.4) can be written in a matrix form as

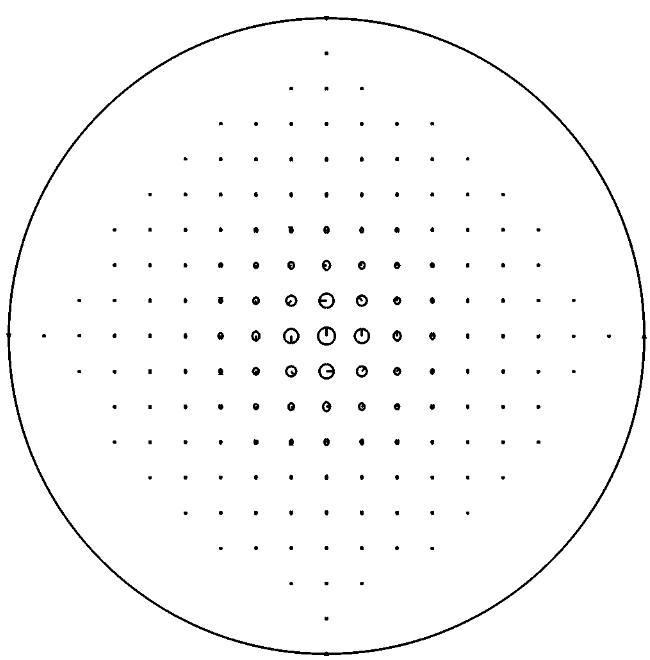

FIG. 2. One of the excited modes, $\omega=0.521$ middle layer.

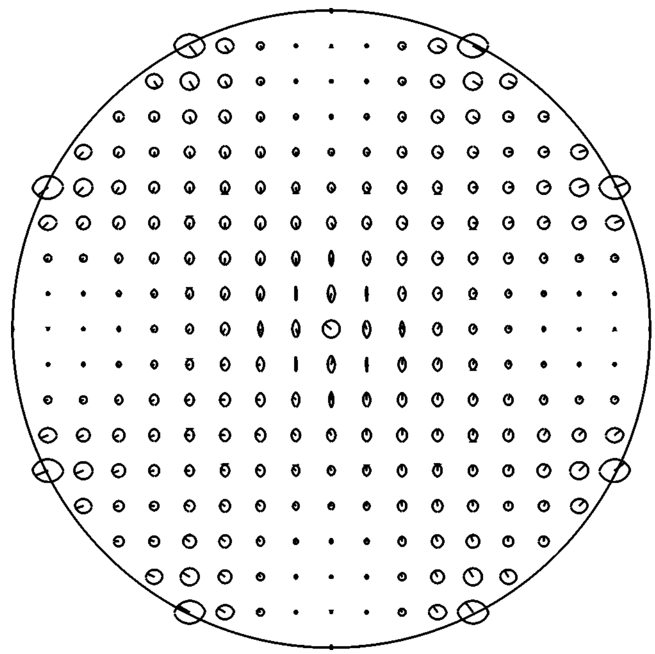

FIG. 3. One of the excited modes, $\omega=4.13$, middle layer.

$$\frac{dm_i^{\alpha(1)}}{dt} = \sum_{j,\beta} A_{ij}^{\alpha\beta} m_j^{\beta(1)} + g_i^{\alpha}$$

$$\hat{\mathrm{x}}^{\alpha} g_i^{\alpha} = \gamma \mathbf{m}_i^{(0)} \mathbf{h}_i^{(\text{rf})} = \gamma \hat{\mathrm{x}}^{\alpha} \sum_{\beta,\chi} \varepsilon^{\alpha\beta\chi} m_i^{\beta(1)} h_i^{\chi(\text{rf})} = \hat{\mathrm{x}}^{\alpha} \sum_{j,\beta} M_{ij}^{(0)\alpha\beta} h_j^{\beta(\text{rf})}. \quad (2.8)$$

Here we used Greek letters to number the coordinates, and Roman letters to number the dipoles, where $A_{ij}^{\alpha\beta}$ is a matrix corresponding to the homogeneous part of Eq. (2.4), and g_i^{α} is an inhomogeneous term corresponding to the applied time-dependent magnetic field. Henceforth it will be convenient to lump the spin index, i, and the vector-component index α into a single index, i'; e.g., we write, $m_i^{\alpha(1)} \to m_{i'}^{(1)}$. Again following the standard procedure to solve a linear ODE, and using our new notation, we can transform Eq. (2.8) into the form

$$m_i^{(1)} = \sum_k P_{ik} y_k, \quad P_{ik} = V_i^{(k)}, \quad P_{ki}^{-1} = V_i^{(k)*},$$

$$P^{-1}P = 1, \quad \frac{dy_k}{dt} = \lambda^{(k)} y_k + \sum_i P_{ki}^{-1} g_i, \quad (2.9)$$

the solution of which is

$$y_k(t) = e^{\lambda^{(k)}t} c_k + e^{\lambda^{(k)}t} \int e^{-\lambda^{(k)}t} \sum_i P_{ki}^{-1} g_i(t) dt. \quad (2.10)$$

We will consider only the case where the applied rf field has a sinusoidal time dependence, in which case the solution can be written as

$$m_i^{(1)}(t) = \sum_{j=1}^{N} \chi_{ij} h_j^{(\mathrm{rf})} e^{-i\omega t}, \tag{2.11}$$

where we introduced the frequency-dependent susceptibility χ_{ij}

$$\chi_{ij} = -\sum_{k=1}^{3N} \frac{\sum_{l=1}^{3N} M_{lj}^{(0)} V_l^{*(k)}}{\omega - \omega^{(k)}} V_i^{(k)}. \tag{2.12}$$

Note that neither the static nor dynamic applied magnetic field needs to be uniform and may have any spatial dependence.

Recalling the thermodynamic expression for the change in magnetic energy,[9] we may write the absorption as

$$\dot{E} = \frac{1}{2}\omega \sum_i \mathrm{Im}[m_i^{(1)} h_i^{(\mathrm{rf})*}]. \tag{2.13}$$

This term can be shown to be proportional to $[h^{(\mathrm{rf})}]^2$; all the quantities occurring in these formulas can be found by using the method given in previous section, which gives

$$\dot{E} = \frac{1}{2}\omega \times \mathrm{Re}\left[-\sum_{k=1}^{3N} \frac{[\sum_{l=1}^{3N}\sum_{j=1}^{3N} m_{lj}^{(0)} H_j^{(\mathrm{rf})} V_l^{*(k)}]\sum_{i=1}^{3N} V_i^{(k)} H_i^{(\mathrm{rf})*}}{\omega - \omega'^{(k)} + i\beta\omega''^{(k)}} \right]. \tag{2.14}$$

This formula can be used to calculate the absorption as a function of both the frequency of the rf field and the strength of the static magnetic field. Note that the quantity

$$c_k = \left[\sum_{l=1}^{3N}\sum_{j=1}^{3N} m_{lj}^{(0)} H_j^{(\mathrm{rf})} V_l^{*(k)} \right] \left[\sum_{i=1}^{3N} V_i^{(k)} H_i^{(\mathrm{rf})*} \right] \tag{2.15}$$

plays the role of an *oscillator strength*—it can be calculated separately and for a given ω then involves only a single sum.

III. ABSORPTION SPECTRUM AND RESONANCE MODES OF A VORTEX STRUCTURE IN A DISK

We now discuss the resonant modes for a vortex ground state. It is convenient to switch to dimensionless units in which the magnetic moments and the gyromagnetic ratio are unity; such units are connected with the Gaussian system by

$$\frac{H_{\mathrm{Gauss}}}{M_s} = H, \quad \frac{\omega_{\mathrm{Gauss}}}{\gamma M_s} = \omega, \quad J = \frac{A_{\mathrm{Gauss}}}{M_s^2 a^2}. \tag{3.1}$$

We treat the case of a body consisting of five layers, each of which has a diameter of 19 (in units of the nearest-neighbor spacing). We have done similar calculations with many more dipoles, but the absorption spectra are similar for the case of a uniform rf field, although there are of course more modes. (Note that the modes characterized by shorter wavelengths are not strongly excited by a uniform rf field.) The axes are chosen such that the disk is lying in an x-y plane, J is equal to 5, and $\beta = 0.005$. The equilibrium configuration is a vortex with its core in the center of each of the layers. In this particular case there is only a small variation in the spatial distribution of the moments among these five layers. One must be careful to find the equilibrium ground state—there are many states with slightly higher energies but with completely different absorption spectra.

We performed calculations with two different applied rf field configurations—both were uniform, but one was parallel to x axis and the other parallel to z axis. The absorption spectrum for the first case is presented in Fig. 1, with some of the excited modes presented in Figs. 2 and 3. The ellipses centered on each site approximately represent the orbits of individual spin precession, and the lines inside represent the relative phases. One should keep in mind that the equilibrium axes around which the dipoles precess differ (note the central dipoles tip out of the plane of the disk, while the remainder lie in plane). The mode presented in Fig. 2 is completely localized within the vortex core. We note that this is somewhat similar to the mode studied previously by Guslienko *et al.*, although the difference in our approaches prevents us from establishing a rigorous connection.

The modes excited by the field applied in disk plane have oscillations localized within the vortex core, and exhibit Bessel-functionlike behavior outside of the vortex core. If the core is removed (by creating a hole in the middle of such a disk), the modes obviously would become doubly degenerate and can be relatively well described by Bessel functions, as discussed in Ref. 10.

IV. CONCLUSIONS

We have developed a method for calculating the resonant modes and the absorption characteristics of a magnetic body of arbitrary shape in an arbitrary static and dynamic magnetic field that is applicable in the linear regime. The techniques have been demonstrated for a vortex ground state in a three-dimensional (3D) disk. This work was supported by NSF Grant No. ECS-0224210.

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